Table 2[a] Generalized 2-valued system

A_1	A_2	$\ldots$	$A_{n_{i-1}}$	A_{n_i}	B_i	P_{ij}
0	0	$\ldots$	0	0	$v_{i,1}$	$P_{A1}P_{A2}\ldots P_{Ani}$
0	0	$\ldots$	0	1	$v_{i,2}$	$P_{A1}P_{A2}\ldots P_{Ani-1}(1 - P_{Ani})$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\vdots$
1	1	$\ldots$	1	1	$v_{i,k}$	$(1 - P_{A1})(1 - P_{A2})\ldots(1 - P_{Ani})$

[a] $k = 2^{n_i}$.

The probability of success of this phase is a function of the success states, namely, the row entries for which $v_{ij} = 0$. It is assumed again for the sake of convenience that there are only two states, denoted by 0 and 1, where 0 means success.

The probability of occurrence of the typical row entry is simply the product of the probabilities of occurrence of the given values of the variables A_1 to A_{ni} for that row. These are the functions P_{ij}, $j = 1,2\ldots 2^{n_i}$, given in the table.

Consequently, the probability of success of the ith mission phase is given as the sum

$$P_i = \Sigma_l P_{i_l}'$$

where l enumerates those P_{ij}'s associated with $v_{ij} = 0$.

Multiple Rocket Firing Unit for Altitude Testing

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THE test firing of small, solid propellant rockets under simulated altitude conditions has generally required test facility occupancy time in excess of 1 hr/motor at Arnold Engineering Development Center (AEDC). It has been necessary to complete the cycle of installing, pumping to altitude, calibrating, firing, returning to ambient conditions, and removing the test article for each motor tested. When a limited number of motors of a type are to be tested, little can be done to reduce the time consumed in completing the cycle. However, when large numbers of nearly identical motors are to be tested, automatic techniques are suggested.

Two of the more time-consuming portions of the small motor testing cycle are the operations of pumping to altitude and, after the firing, returning to ambient conditions. By firing several motors in the test cell during a single pumpdown period, the facility occupancy time can be reduced considerably. This could be accomplished by mounting several motors in the test cell, each attached to a separate load cell, but by so doing a homogeneous sample would be violated in that a common instrumentation system would not be used for all test articles. Such data accuracy refinements as deadweight thrust calibration would also be difficult to achieve.

In order to provide the best possible data while alleviating the problem of high occupancy time when testing large numbers of relatively simple motors, a multiple rocket firing unit (MRFU) has been designed and fabricated at AEDC.

The MRFU is a high accuracy, high testing rate system. It is composed of an electrically controlled and hydraulically operated conveyor mechanism located inside an altitude test cell and various support equipment both inside and outside the test cell (Fig. 1). The present system is capable of testing, during a single pumpdown period, a maximum of 13 motors producing up to 5000-lbf thrust each, at simulated altitudes in excess of 100,000 ft.[1] Also available are 6 deadweight thrust calibration steps and a test article temperature conditioning capability ranging from $-100°$ to $+250°$F.

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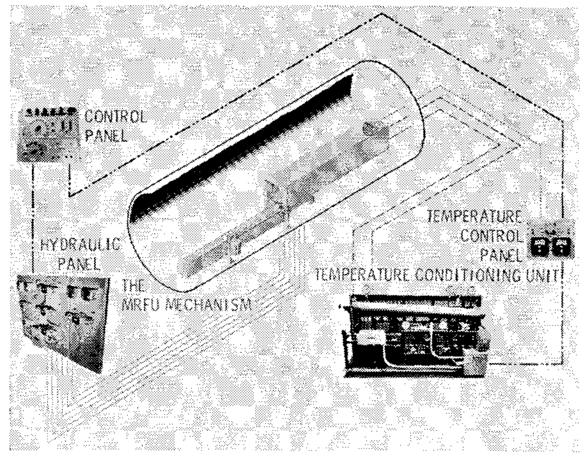

Fig. 1 Schematic diagram of the MRFU system.

Operating Sequence

The test articles are attached to special adapters and mounted nozzle-up between spring-loaded clamping arms along the horizontal conveyor (Fig. 2). A typical operating sequence begins with the conveyor and motors inside the temperature conditioning box. A selection is made at the master control panel, and the conditioning box doors open automatically; the conveyor positions the first motor over the thrust train, and the vertical positioner extends depressing the spline mounted clamping arms, thereby lowering the motor into the firing position. As the motor is lowered, electrical connectors for the firing circuit and motor instrumentation are mated. The motor is locked to the thrust train by hydraulic pressure and isolated thereon by the extension of the isolator wedge, which separates the clamping arms. Data acquisition systems may then be calibrated and the motor fired.

Thrust Train

To keep installation and instrumentation variances at a minimum between test articles, it is desirable to test all articles using highly repeatable installation techniques and common instrumentation. Both these criteria are met by the MRFU thrust train (Fig. 3). The major components of the thrust train are the adapter, locking chuck, thrust calibrator ring, and load cell. The adapter, with the test article attached, is locked to the chuck as hydraulic pressure draws the cone pin down and expands the split head inside the adapter.

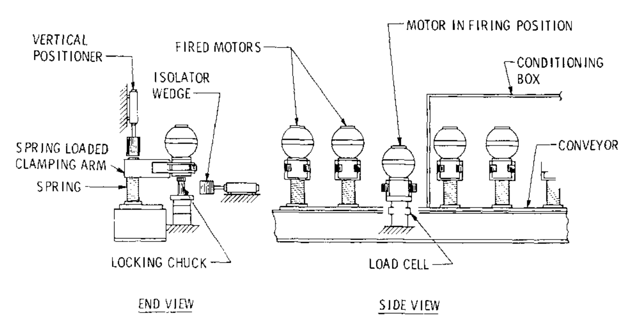

Fig. 2 Schematic diagram of the MRFU mechanism.

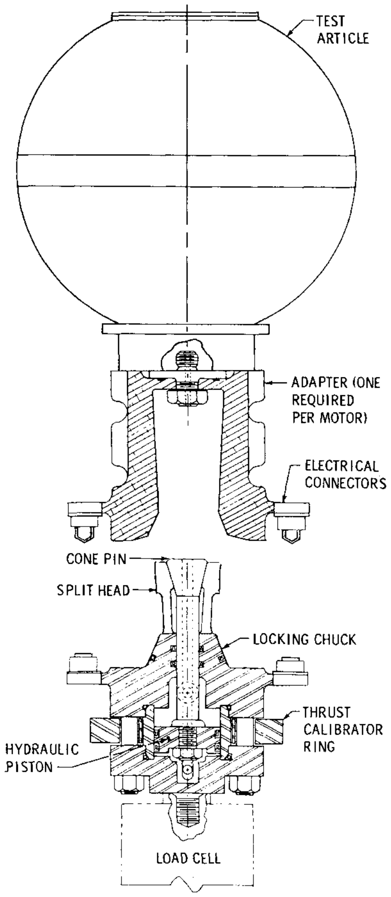

Fig. 3 Section view of the thrust train mechanism.

Small solid propellant rockets usually have short thrust rise times that deliver to the thrust train a hammer-like blow causing thrust train "ringing" or vibrations. Since infinite frequency response thrust trains that can exactly follow a forcing function are not available, designers must resort to thrust trains that are capable of measuring either steady-state thrust or total impulse. Thrust train characteristics necessary for measuring both are not compatible. A high compliance, highly damped, thrust train will accomplish the former, but the more useful parameter for determining rocket performance is total impulse. It is desirable to have a solid, low compliance thrust train to measure with high accuracy the impulse of the test article. Ignition transients in the thrust train cause oscillations in the recorded history which are not characteristic of the rocket motor. However, these oscillations cause very little error in the recorded total impulse of the motor provided the thrust train is a linear second-order oscillatory system such that those half cycles above the true thrust level cancel those below when the integral of the history is taken.

Although the total integral, or impulse, is not harmed significantly by a ringing thrust train exhibiting linear second-order characteristics, discrete thrust points are virtually impossible to determine closer than an estimated axis constructed through the sinusoidal oscillations. If it is assumed that the thrust train oscillations following a step input can be represented by a linear second-order differential equation, then it is possible to solve for any forcing function or rocket thrust that may be applied to the thrust train. By so doing, discrete thrust points are made available. A detailed study and resulting computer program for solving these equations of motion are presented in Ref. 2.

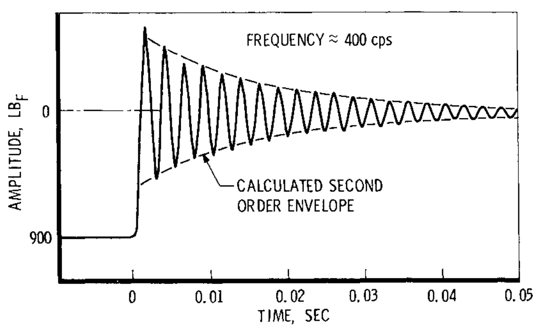

Fig. 4 Dynamic history of the thrust train.

Table 1

	Testing time	Material and man-hour costs
Single firings	1.16 hr	\$1470
MRFU firings	0.57 hr	\$ 597
Savings	50%	60%

To determine the dynamic characteristics of the thrust train, a "twang" test of the system was performed. An 8-lb mass was attached to the adapter, and a 900-lb tension load was applied to the thrust train. The tension was instantaneously released while thrust data acquisition instrumentation was operating. The signal was recorded in analog form on a light-beam oscillograph equipped with a 600-cps frequency response galvanometer. The displacement vs time trace is presented in Fig. 4. The dotted envelope was calculated using linear second-order assumptions with the frequency and damping factor obtained directly from dynamic test data.

Testing Economy

As in the use of most automatic equipment, the savings realized through use of the MRFU for simulated altitude testing are twofold: a reduction in both testing time and the number of personnel required. Seventy-two motors have been test fired on the MRFU, and 23 identical motors tested singularly at AEDC. Table 1 shows a per motor comparison of testing time and costs for these tests. The percentages are quite conservative in that some shake-down problems were experienced during the early portion of the test series.

As crew proficiency grew and final adjustments were made to the MRFU, it was found that a motor could be tested every 10 to 12 min including the time required for pre- and post-fire deadweight calibrations.

Further economy is realized in that the MRFU system was installed in an existing test cell thereby increasing the utilization of the facility.

Data Accuracy

Data accuracy is improved by the high testing rate and highly repeatable installation techniques made possible by the MRFU. By testing a series of articles during a single test cell evacuation period, the environment of the series is assured of being as nearly identical as possible. The short testing time required and the use of the same data acquisition systems for all articles are conducive to low electronic and human variances between samples. These advantages make the system ideal for perturbation or statistical studies.

An error analysis of the thrust system revealed that the data acquisition error to be expected in the total impulse of a test article fired on the MRFU is less than 0.25% (standard deviation) of the total measured impulse.

Conclusions

The multiple rocket firing unit has shown that the inherent complexity of automatic equipment does not preclude such desirable features as a linear second-order thrust train designed to use a common instrumentation system for all test articles, a high accuracy deadweight thrust calibrator, and a test article temperature conditioning capability. In two actual test programs, the MRFU cut testing time and costs in half;

even larger savings are indicated for the future. It is concluded that, when large numbers of small, solid propellant rocket motors are to be tested under simulated altitude conditions, substantial gains in testing economy and data accuracy can be achieved through use of automatic testing equipment.

References

[1] "Rocket test facility," *Test Facilities Handbook* (Arnold Engineering Development Center, 1963), 5th ed., Vol. 2.

[2] Sprouse, J. A. and McGregor, W. K., "Investigation of thrust compensation methods," Arnold Engineering Development Center, AEDC-TDR-63-85 (August 1963).

Pressure-Fed Liquid Rocket Payload Potential

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Nomenclature

c_0 = oxidizer heat capacity, Btu/lb-°F
g = gravitational const (earth), ft/sec^2
I_{sp} = propellant specific impulse in vacuum, sec
k_i = effective thermal conductivity, Btu/hr-ft-°F
R = mixture ratio
t_m = mission time, hours
ΔT_0 = oxidizer temperature rise, °F
ΔT_{0i} = temperature difference, oxidizer to insulation surface, °F
ΔV = velocity increment, fps
W_v = initial gross vehicle weight, lb
W_{pe} = weight of propellant expelled, lb
W_{pr} = residual propellant weight, lb
W_{tkg} = weight of propellant tanks, gas tank, and gas, lb
W_i = insulation weight, lb
α = tankage factor, lb/ft^3
β = propellant payload fraction
δ_v = propellant fraction
η_e = propellant expulsion efficiency
ρ_p = propellant bulk density, lb/ft^3, g/cm^3
ρ_0 = oxidizer density, lb/ft^3, g/cm^3
ρ_f = fuel density, lb/ft^3, g/cm^3
ρ_i = insulation density, lb/ft^3
$\dot{\tau}$ = insulation fraction per unit storage time, h_r^{-1}

Introduction

THE pressure-fed liquid rocket is an extremely versatile design concept. In the lower limit, the design can be as simple as a solid motor for applications requiring a single programed impulse. In the upper limit, it can be designed for any range or sequence of thrust and/or impulse modulation required for the most sophisticated space missions. This note compares the potential payload capabilities of high-energy cryogenic and storable propellant combinations in pressure-fed rockets for spacecraft missions. Payload fractions were determined with respect to mission time and velocity requirements.

Minimal Storage Missions

Comparison of pressure-fed propulsion systems can be made relatively absolute because of the nearly exact equivalency that can be established between systems utilizing different propellant combinations. The detailed evaluation of a multitude of design cases can therefore be circumvented by the device of comparing the payload capabilities on the "common denominator" of equal initial vehicle weight, velocity increment, and propellant storage and expulsion subsystem design criteria. The common design criteria selected are 95% theoretical propellant specific impulse for 100 psia chamber pressure and a 50/1 nozzle expansion ratio, 98% propellant expulsion efficiency, and a tankage factor of 2.8 lb/ft^3 based on spherical tanks operating at a pressure of 150 psia. The propulsion systems employing liquid hydrogen are an exception only to the extent that they optimize at a lower chamber pressure of approximately 70 psia, with corresponding tank pressures of 120 psia.

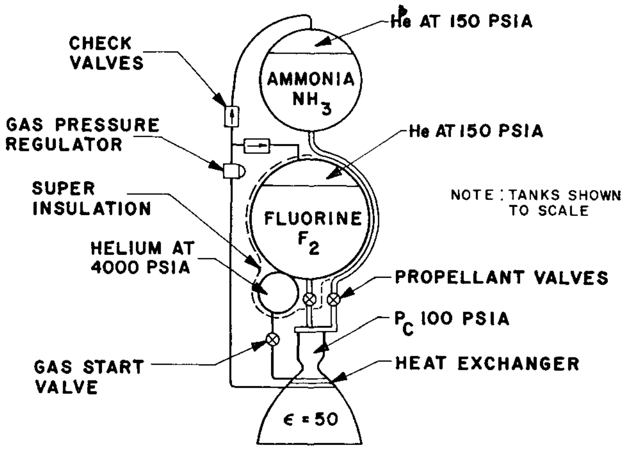

Fig. 1 Pictorial schematic of typical propulsion system.

The tankage factor (α) is the weight of pressurizing gas, gas tankage, and propellant tankage per cubic foot of propellant expelled; 2.8 lb/ft^3 is the potential value for earth-storable propellant systems pressurized with unheated helium and for the cryogenic propellants with helium gas storage at the temperature of the colder propellant and with a heat exchanger to raise the temperature of the delivered helium to that of the warmer propellant. The propellant expulsion efficiency (η_e) of 98% is the over-all value, taking into account the residual propellants (liquid and vapor) resulting from operating mixture ratio error and tank expulsion efficiency. A typical propulsion system designed to this criterion is shown schematically in Fig. 1.

The rocket performance criterion chosen is a propellant payload fraction parameter,

$$\beta \equiv (W_v - W_{pe} - W_{pr} - W_{tkg})/W_v \tag{1}$$

This fraction (β) includes all elements of rocket vehicle weight not primarily determined by the propellant selection, i.e., net payload, guidance system, control system, stage structure, and rocket engine weight. The engine weight is included in β because it is determined primarily by vehicle/flight-plan considerations such as required thrust-to-weight ratio. It can be shown that the definitive Eq. (1) reduces to

$$\beta = 1 - \delta_v[(1/\eta_e) + \alpha/\rho_p] \tag{2}$$

where

$$\delta_v = W_{pe}/W_v = 1 - e^{-x} \qquad x = \Delta V/gI_{sp}$$

Thus β is directly expressed in terms of the propellant properties (I_{sp} and ρ_p), the system design constants (η_e and α), and the mission requirement (ΔV). The conventional specific impulse (I_{sp}) vs mixture ratio (R) data for propellant performance was converted to I_{sp} vs ρ_p data by

$$\rho_p = (R + 1)/[(R/\rho_0) + 1/\rho_f] \tag{3}$$

Figure 2 shows the basic performance of eight propellant combinations for one-day storage time and a ΔV requirement of 16,000 fps. The highest payload line (β) intercepted by

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